Spinoza's concept **"attribute"** in 'de deo' - reconstructional notes on **e1d4**

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" ... sed omnia praeclara tam difficilia quam rara sunt." [$e5\,finis$]

Abstract

This reconstructional working paper on the concept **"attribute"** in the first book ['de deo'] of Spinoza's "ethics" expands the use of the three coordinative definitions, specifying model theoretic analoga for **"attribute"**, **"mode"**, and **"substance"** - which have been introduced in Grafe [8]. Main leverage for this reconstructional project **"attribute"** is, to take Spinoza's definition [**e1d4**][1] literally. Basically in this paper two questions will be considered, starting from **axiom 1** of 'de deo' [for short: **e1a1**], viz., **"omnia, quae sunt, vel in se, vel in alio sunt"**, where Spinoza seems to partition the class of all things in two disjunct classes, the class of things, which are in themselves, and the class of things, which are in other things. This prompts

question (1)

Is this partitioning view (two disjunct non-empty classes of things) compatible with the monist view, that there exists nothing besides (the one) substance in its affections, and if compatible, how ?

The partitioning view is reflected explicitly in George Boole's Spinoza study (-> [1] chapter XIII §§16-18, cf. §17). And a reconstructional answer available shows **question (1)** resolved for the modeltheoretic correlates of **"substance"** and **"mode"** specified in [8][2].

question (2)

If by e1a1 nothing exists besides substance(viz., in it's affections), then what's the ontological status of attributes ?

In my (again reconstructional) answer to **question (2)** I argue, that, in any tenable interpretation of **e1d4**, Spinoza's concept of **"attribute"** need be explicated as **intrinsically relational**.

[1] the label **e1d4** e.g., following a meanwhile in Spinoza scholarship widely shared referencing convention - will be used to refer to definition 4 of part 1 'de deo' of Baruch (Benedict) de Spinoza's "Ethics" (Latin text cited from [13] , Gebhardt pagination in [...], English translation, where explicitly cited, is Curley's [14], free English paraphrasing (often) my own).

[2] with the upshot, that both - the partitioning view and the monist stance - are compatible and hold for the modeltheoretic correlates. This compatability implies a fading away of any supposed ontological relevance of the partitioning view. And this again helps understand the monist view's rational reconstruction enterprise.

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1 introduction

1.1 general remarks

This study of Spinoza's concept **"attribute"** in **e1** aims at sharpening, deepening and extending the use of the three reconstructional coordinative definitions of [8], focussing on the ontological status of **attributes**, with a long-term objective to ***explore some playground for a theory of knowledge within a monist metaphysics approach***.

Alas, the reconstructional approach started with [8] is - to some extent - orthogonal to rather all current discussion of **"attribute"** in **e1**, which has developed in the last decades by international scholars scrutiny[3]. And this reconstructional approach is also toto coelo different in method from the arty

[3]for references see e.g. Noa Shein [11], for a thorough discussion of the "identity puzzle" wrt **substance** and **attribute(s)** see R. Kyle Driggers [5]

set theoretic exercise "suggested by the structure of Spinoza's God" in Joel Friedman[6].

My **main reason to deviate** thus strongly from the above mentioned rich and multifacetted scholarly interpretation-work are then **the reconstructional intention** and **the use of reconstructional means** developed in [8], elaborated to the end, to draw from Spinoza's 'de deo' metaphysics a blueprint for developing and studying a monist metaphysics approach meeting the current state of the art. And Spinoza's 'de deo' imop is a most rewarding starting point for such an enterprise.

In this context a short speculative digression towards guessing

1.2 Spinoza's basic 'de deo' intention or working plan

which imop presumably led to this great architecture of monist metaphysics, and this then is **my respective perception**:

(1) Spinoza sees as the challenge for metaphysical studies in his time the mission, to reconcile (1a) the best of ancient substance metaphysics with the at his time current developments in (1b) the physical science (=philosophy of nature)[4]. And Spinoza is fully aware, that the respective metaphysical approach by Descartes fails to achieve such reconciliation; regarding perhaps as their greatest 'sin' the doctrine of 'created substances' [which might fairly be judged a 'contradictio in adiecto' [5]], and wrt Descartes stress on (non vacuous) metric space Spinoza takes measures by changing the emphasis of **extensio** to **motion and rest**[6], implicitly thence suggesting this metric space being only a derivative from these.

(2) And Spinoza's response to Descartes' poor workmanship wrt (1a) is, to develop an intransigently rigid concept of substance[7], characterized only very general by some "propria of substance". But (wrt 1b) **Spinoza ropes up this metaphysical concept owed to (1a) by his definition of deus**[8] for the integration of the at his time current science **with an in this peculiarity totally new concept of "attribute of substance"**, using "attribute" as the placeholder for the new physics [stating metrical laws and suggesting their experimental, else observational, verification], which in the 17th century is the upcoming perception of nature [=of substance in its affections][9].

[4]Spinoza personally is acquainted e.g. with items of Boyle's and of Huyghen's work (cf. for many Buyse [2]), while the greater scene is given of course by the general development of 'classical' mechanics since Galilei and Descartes.

[5]**e1p7s2** ... vel, si quis statuat substantiam creari, simul statuit ideam falsam factam esse veram, quo sane nihil absurdius concipi potest; ...

[6]cf. Harold. H. Joachim [10], Chapt. III, §2, pp.67-69

[7]descending presumably from some neo-platonist pattern (imaginably Proklos via the Areopagite, or what way ever)

[8]**e1d6** "Per Deum intelligo ens absolute infinitum, hoc est substantiam constantem infinitis attributis, quorum unumquodque aeternam et infinitam essentiam exprimit"

[9]the remaining parallelism wrt "cogitatio" as an independent attribute is another Cartesian heritage, which but Spinoza - still a child of his time - fails to omit. Though, "cogitatio" in Descartes and Spinoza marks the start of what has then been called "rational psychology"

1.3 this paper

The main purpose of this working paper is to elaborate a hopefully faithful but reconstructional interpretation of Spinoza's concept of **"attribute"** in 'de deo', and indicate, how far the formal [model theoretic] analogy, created and presented in [8], complies with this interpretation.

Two primary text sources, to which this paper will refer selectively, are
1. Baruch de Spinoza's Ethica [13], part I 'de deo' (i.e., **e1** for short)
2. George Boole's "laws of thought" [1], Chapter XIII, § 16-18, esp. § 17

1.3.1 remarks on use of reconstructional formalism in this paper

There is some - but very restricted - use of the reconstructional formalism from [8] in answering **question (1)**, to the end
first, to transpose **question (1)**, as it were suggested by Boole's respective analysis, into a contemporary formal setting,
and secondly, to answer the question within that formal setting referring to the reconstructional coordinative definitions.

Quite differently, the part of this paper answering **question (2)** dispenses mostly with reference to formalism, as this section is meant to deliver a detailed motivation and justification for promoting the **relational nature** of **"attribute"** in 'de deo', thereby delivering also a rather comprehensive motivation for the (leading or principal) coordinative definition, viz., the coordinative definition for **"attribute"** as a (large scale) theory, given in [8]. This part of the study is then rather entirely text-based("ethica,.." and "... de intellectus emendatione"), though from a systematic, reconstruction-oriented perspective.

2 spotlights from Boole's discussion of 'de deo'

2.1 on whether Spinoza in 'de deo' attempts to prove the existence of an infinite being

Boole in this chapter studies "portions" of Samuel Clarke's[10] [3] and "portions" of Spinoza's 'de deo' as concurring attempts "to prove the existence of an infinite being"[11]

From my point of view, this interpretation does not really fit in what Spinoza is about in 'de deo'. Instead, wrt this view of the target of 'de deo', I debate Boole's interpretation and follow and expand a respective suggestion of Wolfgang Cramer[12]. Viz., I hold, that Spinoza in 'de deo' does not even intend to prove the existence of an absolute being, but Spinoza's definitions, axioms and

[10] Samuel Clarke is well known from his 'correspondence with Leibniz', there acting as Isaac Newton's mouthpiece

[11] "19. ... It is not possible, I think, to rise from the perusal of the arguments of Clarke and Spinoza without a deep conviction of the futility of all endeavours to establish, entirely a priori, the existence of an Infinite Being, His attributes, and His relation to the universe. ..." [1], pp. 216 f.

[12] [4], chapt. I.c " ... (Substanz)", p.22, pp.26f. , and I.d " ... (Attribut), pp.30 ff.

proofs in 'de deo' start from this existence which is presupposed as unquestionably evident. Cf. for this that **e1p11d** is referring back to **e1p7**, and **e1p7** is referring back to **e1d1**.

Cramer emphasizes that **e1d1** is a "real definition", else that "Definition 1 ist der in Form einer Definition gesetzte ontologische Gottesbeweis"([4], chapt. 2.a, p. 13). This exceptional construct then seems to be the justification, to place the definition of **causa sui** first of all definitions as the very start of 'de deo'.

And, just one more note on **e1p11d**:

though Spinoza mentions additionally some variant of the ontological ("aliter") and as well of the cosmological ("aliter") argument, this is done only as a concession to those not already getting the point of **e1d1**. This is also obvious from the fact, that these replays of the ontological and of the cosmologial argument in **e1p11d** only follow the "concipi,si fieri potest" salvatory clause "Demonstratio . Si negas, concipe, si fieri potest, Deum non existere. ...", and the scholium **e1p11ds** confirms explicitly

"Scholium. In hac ultima demonstratione Dei existentiam a posteriori ostendere volui, ut demonstratio facilius perciperetur; non autem propterea, quod ex hoc eodem fundamento Dei existentia a priori non sequatur. Nam, cum posse existere potentia sit, sequitur, quo plus realitatis alicujus rei naturae competit, eo plus virium a se habere ut existat; adeoque ens absolute infinitum sive Deum infinitam absolute potentiam existendi a se habere, qui propterea absolute existit. ..." [13],p. [54].

"Schol.: In this last demonstration I wanted to show God's existence a posteriori, so that the demonstration would be perceived more easily—but not because God's existence does not follow a priori from the same foundation. For since being able to exist is power, it follows that the more reality belongs to the nature of a thing, the more powers it has, of itself, to exist. Therefore, an absolutely infinite Being, or God, has, of himself, an absolutely infinite power of existing. For that reason, he exists absolutely. ..."[14], p. 556

Starting then from this unquestionable existence of **causa sui**, Spinoza in 'de deo' develops his propositions [ordine geometrico] to show the properties and the structure of this unconditional being. In more detail: the **"propria"**[13] give the necessary properties of the one **"substance"** as part of the description of natura naturans, whereas, on the other hand, each **"attribute"** gives the **substance in its affections**, i.e. its **modi** (the description of natura naturata as perceived by an intellect).

2.2 'de deo' disjunctions in "laws of thought" chapt. XIII §17

Notwithstanding the above remarks, I'd like to refer to Boole's lucid logical apprehension of part of Spinoza's text as the background, to formulate this pa-

[13]Spinoza sums up the propria of the **"substance"** in **e1app**,the appendix of 'de deo', [13] p. [78]

per's **question (1)** and to this end suggest to draw attention to a special part of George Boole's [1] discussion of Spinoza's 'de deo' in "laws of thought" , chapter XIII,§§ 16-18, viz., to §17:

" ...
17 I conceive, then, that in the course of his demonstration, Spinoza effects several parallel divisions of the universe of possible existence, as,

1st. Into things which are in themselves, $\mathbf{x}$, and things which are in some other thing, $\mathbf{x}'$, whence, as these classes of thing together make up the universe, we have

$$\mathbf{x} + \mathbf{x}' = \mathbf{1} \text{ ; (Ax. i.)}$$

or, $$\mathbf{x} = \mathbf{1} - \mathbf{x}' \text{ .}$$

2nd. Into things which are conceived by themselves, $\mathbf{y}$, and things which are conceived through some other thing, $\mathbf{y}'$ whence

$$\mathbf{y} = \mathbf{1} - \mathbf{y}' \text{ . (Ax. II.)}$$

3rd. Into substance, $\mathbf{r}$, and modes, $\mathbf{r}'$; whence

$$\mathbf{r} = \mathbf{1} - \mathbf{r}' \text{ . (Def III.V.)}$$

4th. Into things free, $\mathbf{f}$, and things necessary, $\mathbf{f}'$ whence

$$\mathbf{f} = \mathbf{1} - \mathbf{f}' \text{ . (Def VII.)}$$

5th. Into things which are causes and self-existent, $\mathbf{e}$, and things caused by some other thing, $\mathbf{e}'$ whence

$$\mathbf{e} = \mathbf{1} - \mathbf{e}' \text{ . (Def I. Ax. VII.)}$$

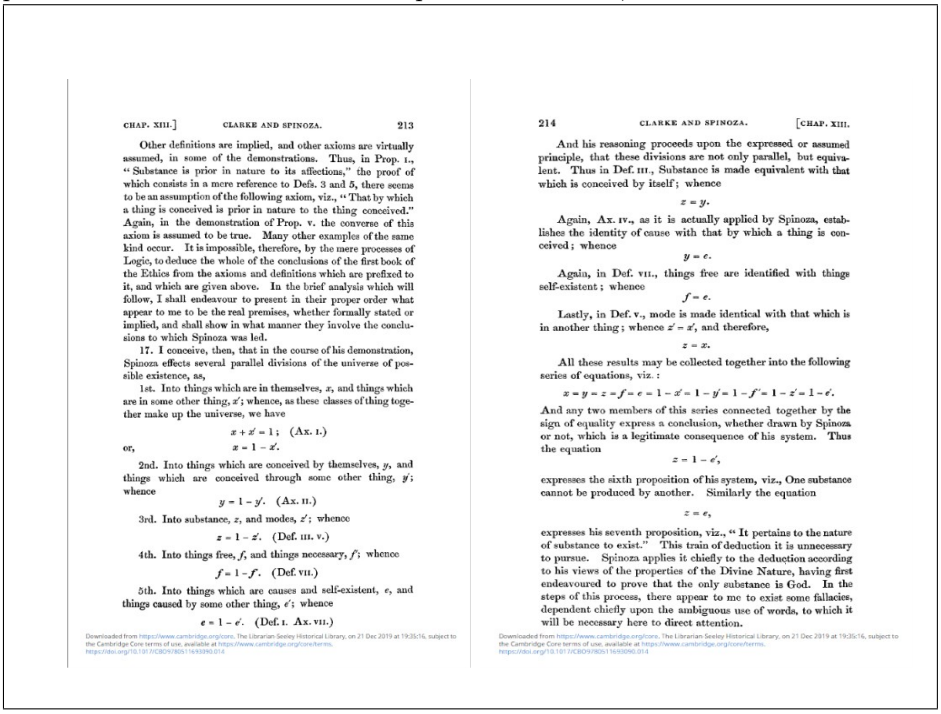
... ... " [1], p.213

Hence Boole, in his highly rewarding analysis of Spinoza's way of argument, symbolizes and lists these five 'de deo' "divisions" of "the universe of possible existence" with a lot of instructive comments including the observation:

"And his [sic: Spinoza's] reasoning proceeds upon the expressed or assumed principle, that these divisions are not only parallel, but equivalent[14]. Thus in Def. III, Substance is made equivalent with that which is conceived by itself; whence $\mathbf{z} = \mathbf{y}$. ..." [1], p.214

[14]equivalent for reason, that the paralleled concepts are taken to be pairwise coextensional

Figure 1: Boole "laws of thought", chapter XIII, §17 : ... Spinoza effects several parallel divisions of the universe of possible existence, ...



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Other definitions are implied, and other axioms are virtually assumed, in some of the demonstrations. Thus, in Prop. I., "Substance is prior in nature to its affections," the proof of which consists in a mere reference to Defs. 3 and 5, there seems to be an assumption of the following axiom, viz., "That by which a thing is conceived is prior in nature to the thing conceived." Again, in the demonstration of Prop. V. the converse of this axiom is assumed to be true. Many other examples of the same kind occur. It is impossible, therefore, by the mere processes of Logic, to deduce the whole of the conclusions of the first book of the Ethics from the axioms and definitions which are prefixed to it, and which are given above. In the brief analysis which will follow, I shall endeavour to present in their proper order what appear to me to be the real premises, whether formally stated or implied, and shall show in what manner they involve the conclusions to which Spinoza was led.

17. I conceive, then, that in the course of his demonstration, Spinoza effects several parallel divisions of the universe of possible existence, as,

1st. Into things which are in themselves, x, and things which are in some other thing, x'; whence, as these classes of thing together make up the universe, we have

$$x + x' = 1 ; \quad \text{(Ax. I.)}$$

or,

$$x = 1 - x'.$$

2nd. Into things which are conceived by themselves, y, and things which are conceived through some other thing, y'; whence

$$y = 1 - y'. \quad \text{(Ax. II.)}$$

3rd. Into substance, z, and modes, z'; whence

$$z = 1 - z'. \quad \text{(Def. III. V.)}$$

4th. Into things free, f, and things necessary, f'; whence

$$f = 1 - f'. \quad \text{(Def. VII.)}$$

5th. Into things which are causes and self-existent, e, and things caused by some other thing, e'; whence

$$e = 1 - e'. \quad \text{(Def. I. Ax. VII.)}$$

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And his reasoning proceeds upon the expressed or assumed principle, that these divisions are not only parallel, but equivalent. Thus in Def. III., Substance is made equivalent with that which is conceived by itself; whence

$$z = y.$$

Again, Ax. IV., as it is actually applied by Spinoza, establishes the identity of cause with that by which a thing is conceived; whence

$$y = e.$$

Again, in Def. VII., things free are identified with things self-existent; whence

$$f = e.$$

Lastly, in Def. V., mode is made identical with that which is in another thing; whence $z' = x'$, and therefore,

$$z = x.$$

All these results may be collected together into the following series of equations, viz.:

$$x = y = z = f = e = 1 - x' = 1 - y' = 1 - f' = 1 - z' = 1 - e'.$$

And any two members of this series connected together by the sign of equality express a conclusion, whether drawn by Spinoza or not, which is a legitimate consequence of his system. Thus the equation

$$z = 1 - e',$$

expresses the sixth proposition of his system, viz., One substance cannot be produced by another. Similarly the equation

$$z = e,$$

expresses his seventh proposition, viz., "It pertains to the nature of substance to exist." This train of deduction it is unnecessary to pursue. Spinoza applies it chiefly to the deduction according to his views of the properties of the Divine Nature, having first endeavoured to prove that the only substance is God. In the steps of this process, there appear to me to exist some fallacies, dependent chiefly upon the ambiguous use of words, to which it will be necessary here to direct attention.

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3 reconstructional approach

3.1 basics and motivation

Grafe [8] stipulates three coordinative definitions for the **e1** concepts **"substance"**, **"mode"**, **"attribute"**, in other words, starts a partial 1-1 mapping

{ **concepts of e1** } ↦ { **concepts of modeltheory** }[15]

The idea of this partial mapping, and - in fact - its validation as a tool for **e1** rational reconstruction is, to show significant similarity of interrelations of central concepts in 'de deo' with the interelations of the coordinated concepts in standard model theory, i.e., the mapping of the 'de deo' concepts is created and expected to provide a non trivial homomorphism from concepts 'de deo' to concepts of model theory. And a key point to achieve this had been found and developed with the mapping for **"mode"** based on the perhaps somewhat unexpected but imop not only convincing but even forced, mapping for the 'de deo' concept **"attribute"** to the model theoretic concept **theory of a relational structure**

The more general idea then is to find thence prospects and/or guidance for a systematic elaboration of an enduring rationale of Spinoza's **e1** = 'de deo'

[15] "modeltheory" in this paper always (unless explicitly noted otherwise) refers as usual to the set theoretic interpretation of standard systems of first order logic

metaphysics by developing and considering corresponding constructs in model theory and reframing (some of the) most difficult interpretation questions of 'de deo' as questions wrt the logical interrelations of the corresponding model theoretic constructs; in the hope, that significant answers can be found there and can - by inverse mapping - be applied to the original, viz., to 'de deo'.

3.2 formalism

The short form of the formalism used for the coordinative definitions, is:

consider an **attribute** - e.g."extension" - given as pair, consisting of a (large scale) theory **Th** (formalized in first order logic), and its **intended model** $< \mathbf{U}, \mathbf{R}_\alpha >$, then

for each theorem $\mathbf{\Theta}$ of theory **Th** wrt the intended model of **Th**, $< \mathbf{U}, \mathbf{R}_\alpha >$,

1. select a **prenex normal form $\mathbf{\Gamma}$ of $\mathbf{\Theta}$**, viz., $\mathbf{pnf}(\mathbf{\Theta}) = \mathbf{\Gamma} = \mathbf{QP}_{\mathbf{\Gamma}} {}^\frown \mathbf{M}_{\mathbf{\Gamma}}$, where $\mathbf{QP}_{\mathbf{\Gamma}}$ is the quantifier Prefix, and $\mathbf{M}_{\mathbf{\Gamma}}$ the quantifier free Matrix of the selected prenex normal form $\mathbf{\Gamma}$ of $\mathbf{\Theta}$.

2. now, for each theorem $\mathbf{\Theta}$ of **Th** :

the model theoretic object representing a **mode** perceived by this **attribute** is the value of its correlated **pnf**-Matrix $\mathbf{M}_{\mathbf{\Gamma}}$, viz., $\mathbf{V}_{\mathbf{M}_{\mathbf{\Gamma}}}$, in the model $< \mathbf{U}, \mathbf{R}_\alpha >$

3. the model theoretic object representing the **substance** perceived by this **attribute** then is the union of the sets $\mathbf{V}_{\mathbf{M}_{\mathbf{\Gamma}}}$ of the **pnf's** matrices $\mathbf{M}_{\mathbf{\Gamma}}$ of all theorems $\mathbf{\Theta}$ of theory **Th**.

For motivation, background and more technical detail of this construct see [8] and [7], for a mini-example for a **mode** from celestial mechanics see [9], section 6.2[16].

3.3 overview and comment

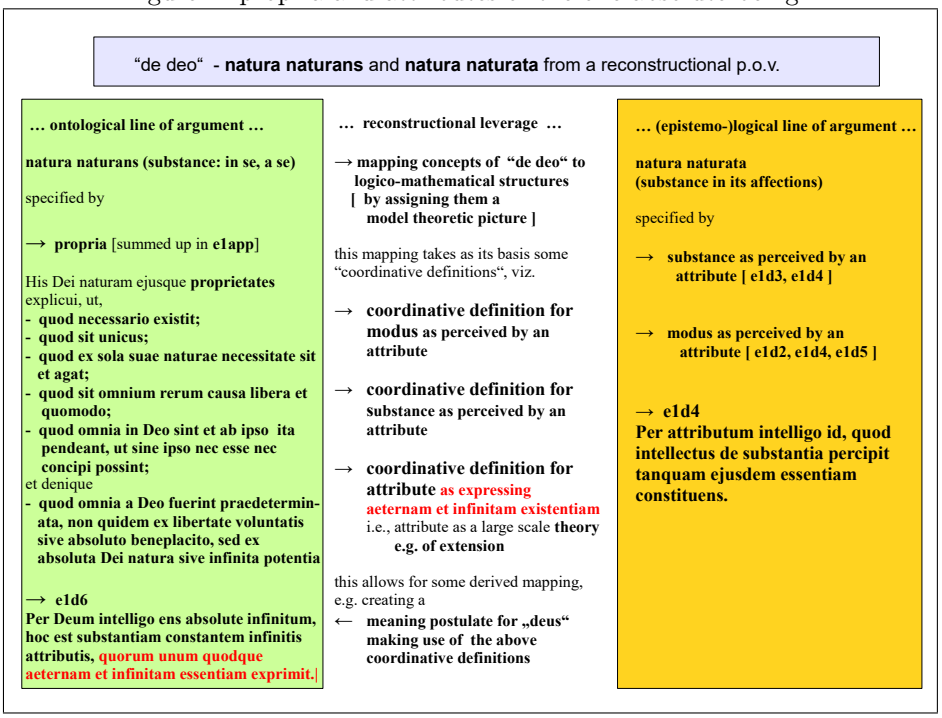
The **e1** concept **"attribute"** (**e1d4**) maps to (i.e., has as it's model theoretic corrrelate) the concept **"theory (first order) of a relational structure"** in the model theoretic standard sense, ([8], §4.1, p.12).

This coordinative definition **and a to this end newly elaborated semantic consequence of the first order logic "prenex normal form"- theorem** allow create a seminal coordinative definition for the **e1** concept **"mode"**, as perceived by an **"attribute"**([8] §4.2, p.12f.), and thence derivatively for a coordinative definition of the **e1** concept **"substance"**, as perceived by an **"attribute"** ([8], §4.3,, p.14f)

An important aspect to keep in mind were, that - in this reconstructional scenario - for any **attribute** the model theoretic picture for each **mode** is determined uniquely up to logical equivalence by the model theoretic picture of the **attribute**, which may be thought of as picturing **a clear and distinct perception of a mode by an attribute**. And as a consequence, the unique determination of the **mode**-pictures implies the unique determination of the **substance**-picture. Taking the inverse mapping gives us the not unexpected but plausible unique determination of **substance, as perceived by an attribute** by this **attribute**.

[16] the example is given there in the context of discussing ontological commitment mitigation, but it applies 1-1 as well to **mode**. The reason for this close connection of topics has been dealt with at some length in [8] and [7]

Figure 2: propria and attributes of the one absolute being


"de deo" - natura naturans and natura naturata from a reconstructional p.o.v.
… ontological line of argument …
natura naturans (substance: in se, a se)
specified by
→ propria [summed up in e1app]
His Dei naturam ejusque proprietates explicui, ut,
- quod necessario existit;
- quod sit unicus;
- quod ex sola suae naturae necessitate sit et agat;
- quod sit omnium rerum causa libera et quomodo;
- quod omnia in Deo sint et ab ipso ita pendeant, ut sine ipso nec esse nec concipi possint;
et denique
- quod omnia a Deo fuerint praedetermin-ata, non quidem ex libertate voluntatis sive absoluto beneplacito, sed ex absoluta Dei natura sive infinita potentia
→ e1d6
Per Deum intelligo ens absolute infinitum, hoc est substantiam constantem infinitis attributis, quorum unum quodque aeternam et infinitam essentiam exprimit.
… reconstructional leverage …
→ mapping concepts of "de deo" to logico-mathematical structures [by assigning them a model theoretic picture]
this mapping takes as its basis some "coordinative definitions", viz.
→ coordinative definition for modus as perceived by an attribute
→ coordinative definition for substance as perceived by an attribute
→ coordinative definition for attribute as expressing aeternam et infinitam existentiam i.e., attribute as a large scale theory e.g. of extension
this allows for some derived mapping, e.g. creating a
← meaning postulate for „deus" making use of the above coordinative definitions
… (epistemo-)logical line of argument …
natura naturata (substance in its affections)
specified by
→ substance as perceived by an attribute [e1d3, e1d4]
→ modus as perceived by an attribute [e1d2, e1d4, e1d5]
→ e1d4
Per attributum intelligo id, quod intellectus de substantia percipit tanquam ejusdem essentiam constituens.

4 question (1) resolved in the model theoretic picture

4.1 question (1)

Boole's logical observations wrt Spinoza's "disjunctions" provide a rather instructive embedding for **question (1)** of this paper, i.e., for the question:

Is this partitioning view (two disjunct non-empty classes of things) compatible with the monist view, that there exists nothing besides (the one) substance in its affections, and if compatible, how ?

4.2 answer & explanation

In order to answer the question, I transpose Boole's hitherto "disjunctions" result to the model theoretic reconstructional setting, where an answer is easily available. The only formal means needed for this transposition is use of the *powerset algebra* of a given set, i.e., the meanwhile commonplace use of set theoretic inclusion, intersection and union for a given countable (finite or countably infinite) set[17].

Now, agreeing with Boole's already mentioned observation of the equivalence of Spinoza's "disjunctions", in order to answer question (1) it's sufficient to consider the parallel between the first and the third disjunction:

[17] just this formal means belongs to the core heritage, we owe to George Boole. Hence this transposition is also historically justifiable

1st. Into things which are in themselves, „, , and things which are in some other thing, ...
and
3rd. Into substance, ... , and modes ...

As already noted above in the paragraph on formalism:
for a given model theoretic picture of an **attribute** the model theoretic picture of **substance, as perceived by this attribute** is defined as the set theoretical union of the model theoretic pictures of all **modes, as perceived by this attribute**.

Accordingly in the model theoretic picture we have both:
First, as in the model theoretic picture, **substance** is a set, the power set of **substance** exists, and as well **substance** itself, and all its **modes** are mutually different elements of this powerset.
Secondly, this great multiplicity has but no ontological import, because the model theoretic picture of **substance** contains the model theoretical pictures of all **modes**, though not as elements, but as subsets.

Summing up, the answer to question (1) in the model theoretic picture is to the positive:
although there is a multiplicity in the universe of discourse counting **substance** and **modes** as *many*, this multiplicity is only a set theoretical epiphenomenon of the power set algebra over the model theoretic picture of **"substance"**, which is but defined as the set theoretical union of the **modes**, thus the monist stance remains untouched by the 'de deo' disjunctions.
Now, my further suggestion, that the answer found for the model theoretic pictures of **"substance, as perceived by an attribute"** and **"mode, as perceived by an attribute"** - by inverse mapping - applies as well for the question wrt the original concepts 'de deo', relies on the conviction that the model theoretic picture of **modes**, as drawn via a specific semantic definition, descending from the model theoretic picture of **attribute** preserves metaphysical structure.
There can't be a direct proof of this kind of adequacy. **Eventually the whole reconstructional enterprise need be judged a) from the consistency of it premisses, and b) from the fertility of its application.**

5 question (2) - the ontological status of attribute

e1d4 : Per attributum intelligo id, quod intellectus de substantia percipit tanquam ejusdem essentiam constituens.

e2p1 : Cogitatio attributum Dei est, sive Deus est res cogitans.
cave: deus sive natura, cf. e.g. e4.praefatio[18]

e2p2 : Extensio attributum Dei est, sive Deus est res extensa.

[18] " ... illud ... infinitum ens, quod Deum seu Naturam appellamus, ..." [13] p. [207]

5.1 parallelism of res cogitans and res extensa

As already hinted to in the introduction - from the reconstructional p.o.v. the parallelism of "cogitatio" and "extensio" as independent attributes is another Cartesian heritage in 'de deo' which is, from a contemporary glance at neurophysiology and it-technology including AI, totally outdated and imop of no current systematic interest. But of course, in referring to the way of argument in 'de deo', e.g. when dealing with **an intellect's perception of natura naturata by the attribute of extension** , Spinoza's differentiating between the two attributes can't be neglected, but the current reconstructional approach does neither rely on nor contradict respective problematic theses, e.g. of 'limiting' (**e1d2**), or of cross-attribute-identification of modes(**e2p13**).

5.2 question (2)

The positive answer to question (1) in the preceding section - viz., that the Spinozean disjunctions, elaborated and emphasized by Boole in his "laws of thought" chpt. XIII, §17, do not mitigate the strictly monist thesis, that nothing exists besides **"substance"** (but) in its affections(= in it's **modes**) - then replays the incisiveness of

question (2)
If by e1a1 nothing exists besides substance(viz., in it's affections), then what's the ontological status of attributes ?

Well, the answer to this question is given by Spinoza manifestly in **e1d4**, the definition of **"attribute"** with an add on in **e1d6**. And I guess, this answer has been somewhat underrated by commenting scholars in the field, and therefore I like suggest revisiting **e1d4** terms and details.

My interpretation path then starts taking the term **"intellect"** in the definiens of **e1d4** as referring to a **finite** and/or **infinite mode** of **substance** perceived by the **cogitatio attribute**:

5.3 e1d4: the role of intellect

Perhaps somewhat unexpectedly, **"intellect"** is not defined in the 'ethics', and while **"intellectus infinitus"** occurs rather often in Spinoza's proofs, but in places where one might expect the term **"intellectus finitus"**, we read [in the later parts of the ethics] regularly **"mens humana"**, but this latter term seems in Spinoza a weaker concept, allowing for **mens humana** deficiencies in perception.

In order to revisit some more detail I start with two of the rare text passages, where Spinoza's gives a metaphysical placement for the **"intellectus finitus"**:

[i1] **e1p30**
"Propositio XXX. Intellectus, actu finitus aut actu infinitus, Dei attributa Deique affectiones comprehendere debet et nihil aliud."

which reserves for **actu intellectus finitus** highest epistemic performance, i.e. a mens humana able to grasp the essence of substance[19].

[i2] the start of the corollary to **e2p11c**
" Corollarium. Hinc sequitur mentem humanam partem esse infiniti intellectus Dei; ..."
which might be taken to mean something like "the human mind is part of the self-perception of the universe".

But these two qualifications - though comprehensible and in their setting appropriate - do not give any detailed information wrt the mode of operation of the **intellect**, finite or infinite. For this latter kind of information we have to look for another source, viz., the "Tractatus de intellectus emendatione". There, from the second part of the treatise ([12] §§91 ff. (=pp. 83 ff.)) we learn:

[i3] that **the role of the intellect is to form a clear and distinct idea of the *res* under consideration**:

"... Scopus itaque est claras et distinctas habere ideas, tales videlicet, quae ex pura mente et non ex fortuitis motibus corporis factae sint. ..." [12] §91, [20]

[i4] **And wrt substance as a causa sui the required idea need be the idea of its essence (and of nothing else)**[21]

"... Quoad primum, ut jam tradidimus, requiritur ad nostrum ultimum finem, ut res concipiatur vel per solam suam essentiam, vel per proximam suam causam. Scilicet si res sit in se, sive, ut vulgo dicitur, causa sui, tum per solam suam essentiam debebit intelligi; ..." [12] §92

[i5] **And there won't/can't be clear and distinct ideas of mutable single things**

"... Sed notandum, me hic per seriem causarum et realium entium non intelligere seriem rerum singularium mutabilium, sed tantummodo seriem rerum fixarum aeternarumque. ..." [12] §100

[i6] Summing up: for a **causa sui-*res*** an adequate idea specifies its essence, available for the **infinite intellect**, while the case for **mens humana** depends[22]

[19] Shein [11], section 1.8, p.13 complains ambiguity: "“By attribute I understand what the intellect perceives of substance as constituting its essence” (1D4).[12] The first term that is ambiguous is “intellect,” since it can refer either to the finite intellect or the infinite one (cf. diagram in Section 1)." But, quite obviously, **e1p30** but gives us, that there is no ambiguity, but (partial) inclusion

[20] cf. **e2d3, e2d4**

[21] cf. **e1d4, e1d6**

[22] **e1p30**, end of **e2p11c** "... tum dicimus mentem humanam rem ex parte sive inadaequate percipere."

5.4 e1d4: the role of attribute

Each **attribute** - by **e1d6** expresses an imperishable and infinite essence of **substance**.

Now, in order to do this job *neither* the **attribute** can be identical with **substance**, *nor* can it be a structural similar copy of an essence of **substance** which existed previous to and independent of the **attribute**; for a relevant passage - in **e2p5d** - see below *[a7]*. ,

5.4.1 digression on "identity" theses

The *neither* clause above comments on explications of **attribute**, which promote some kind of respective *"identity thesis"*, e.g. Shein [11] [sections 1.4 and 1.9.3] takes it for granted, that Spinoza in 'de deo' asserts this identity, and as evidence for this he cites from **e1p4d, e1p19, e1p20c2**. To be fair, neither **e1p19** nor **e1p20c2** contain anything decisive to such an identity thesis, but we need have a closer look at a critical clause in **e1p4d**. The complete text passage reads:

*"**Propositio IV.** Duae aut plures res distinctae vel inter se distinguuntur ex diversitate attributorum substantiarum,vel ex diversitate earundem affectionum.*
***Demonstratio.** Omnia, quae sunt, vel in se, vel in alio sunt (per axiom. 1.), hoc est (per defin. 3. et 5.), extra intellectum nihil datur praeter substantias earumque affectiones. Nihil ergo extra intellectum datur, per quod plures res distingui inter se possunt praeter substantias sive, quod idem est (per defin. 4.), earum attributa earumque affectiones. Q. e. d."* **e1p4 - e1p4d** [13], p. [47-48]

and extract the critical clause:
... per quod plures res distingui inter se possunt praeter substantias sive, quod idem est (per defin. 4.), earum attributa earumque affectiones ...

Well, obviously Shein reads this *idem* as relating to *substantias* and *earum attributa*, in which case one but would expect not *idem est*, but *idem sunt*.

My reading - instead - of this clause goes:
... quod plures res distingui inter se praeter substantias ... idem est ... ... quod plures res distingui inter se praeter substantiae attributas ... [bc attributes express their respective essences]

I take such identity theses, of Shein's or whatever other coinage, to be not only non-explicative wrt **e1d4** and therefore question begging, but the more from an epistemological p.o.v., to commit a category mistake. For a related category mistake see the closing section of [7].

5.4.2 reconstructional theses on attribute

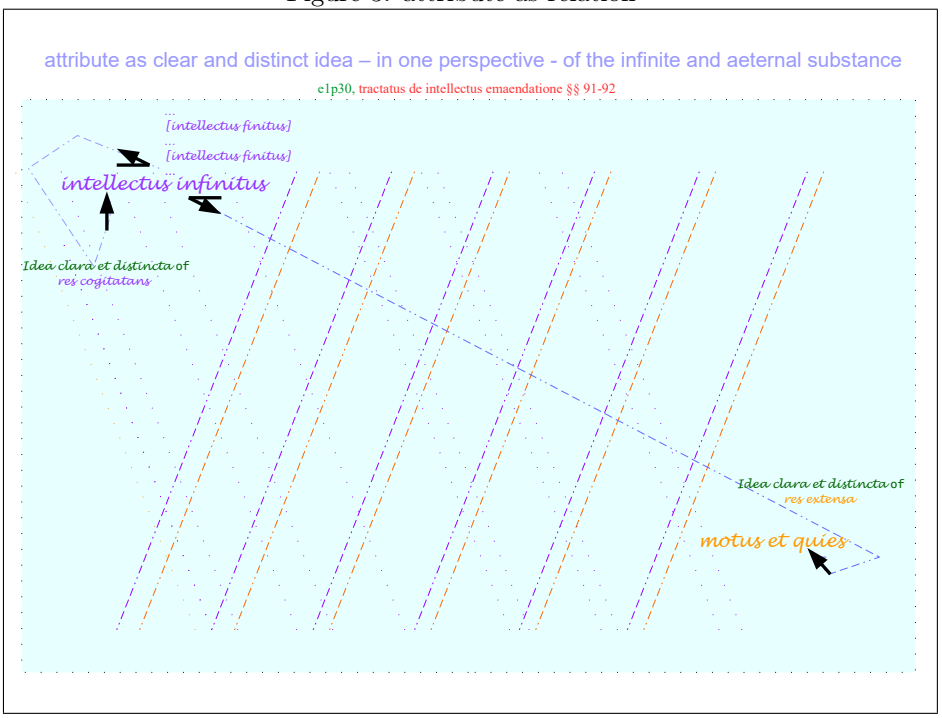
Now lets return to the reconstructional topic. Subsequent to the preceding explorations for **"intellect"** I hold the following to be correct and reconstruction complying theses on **"attribute"**

[a1] **an attribute is a clear and distinct idea of the one causa sui (=substance=god), formed by a finite or the infinite intellect, expressing (in this idea's perspective) infinite and eternal essence.**
[- from*[i1, i3, i4, i6]*].

[a2] this is understood to imply, that there would not exist an **attribute** of **substance**, if there were no **intellect** [23].
[- from [a1]]

[a3] hence an **attribute** is a *many-one-relation* between a finite or the infinite **intellect** as the one relatum, and the basic infinite mode of **substance** as the other relatum. More closely:
[a31] the attribute of *res cogitans* is a *many-one-relation* between a **finite** or the **infinite intellect** as the one relatum, and the infinite mode **infinite intellect** as the other relatum.
[a32] the attribute of *res extensa* is a *many-one-relation* between a finite or the infinite **intellect** as the one relatum, and the infinite mode **motus et quies** as the other relatum.
[- from*[a1, i2, i4, i5]*].

Figure 3: attribute as relation


attribute as clear and distinct idea – in one perspective - of the infinite and aeternal substance
e1p30, tractatus de intellectus emaendatione §§ 91-92
[intellectus finitus]
[intellectus finitus]
intellectus infinitus
Idea clara et distincta of
res cogitatans
Idea clara et distincta of
res extensa
motus et quies

[23]but as **cogitatio** belongs to the **attributes of substance**, this case won't occur

/a4/ Nevertheless, an **attribute** of **substance**, as an idea formed by an intellect (see above */a1/*), is an objective reality, but this objective existence is the existence of an **internal relation** between modes of substance, as described above in /a1, a3].
[- importantly cf. **e1d6** contributing significantly to this point of objective reality of the attributes, recall from above figure 2: propria and attributes of the one absolute being] .

/a5/ This relation must be an internal relation, in the sense that every relation between modes is internal wrt **substance in its affections/modes**.
[- from*/a3, a4/*].

/a6/ **attribute** is only one type of internal relations of **substance** besides other types of internal relations, e.g. there are 'causa' relations between different **infinite modes** perceived by the same **attribute**
[- cf. e.g. **e1p23**].

/a7/ **every attribute is an active creation of the infinite intellect**,
[- this follows rather explicitly from the combination **e2d3, e2p3, e2p5d**]

e2p5d "Demonstratio . Patet quidem ex propositione 3. hujus. Ibi enim concludebamus Deum ideam suae essentiae et omnium, quae ex ea necessario sequuntur, formare posse ex hoc solo, nempe quod Deus est res cogitans, et non ex eo, quod sit suae ideae objectum ... Esse formale idearum modus est cogitandi (ut per se notum), ..." [13] p. [88], translated as "Dem.: This is evident from P3. For there we inferred that God can form the idea of his essence, and of all the things that follow necessarily from it, solely from the fact that God is a thinking thing, and not from the fact that he is the object of his own idea ... The formal being of ideas is a mode of thinking (as is known through itself) ..." [14] p. 591 .

5.5 summing up then

I claim, that **the reconstructional coordinative definition for "attribute"** in [8], **assigning a first order theory** of a relational structure, which is the intended model of that theory, **as the modeltheoretic picture of an "attribute"**,

is fully compatible with the above interpretation, and

complies with the relational character of "attribute, and

allows for the 'de deo' qualification
" ... infinitis attributis, quorum unumquodque aeternam et infinitam essentiam exprimit" .

Now, hoping to increase somewhat further the plausibility of the above scenario, I close this papers considerations by sketching a little

5.6 example

And here it is:

a finite intellect X, considers some part of celestial kinematics under the attribute of extension, viz., X perceives Kepler's second law

"A line joining a planet and the Sun sweeps out equal areas during equal intervals of time."[15], p.5

Let's look for some detail:

X has at his command at one hand
Y1 = a special kinematic theory (viz., a theory that covers metric trajectory changes in metric time intervals[24], which is applicable to positions of planets relative to the sun of our local solar system, and includes as an assumption already Kepler's first law:

"The orbit of every planet is an ellipse with the sun at one of the two foci."[15], p.3

and further X has at his command a set of verified astronomical data Y2 = a set of "time tables" of planet positions [relative to the sun],

and X concludes from these sources Y1 and Y2 that they fit the hypothesis that Kepler's second law applies to the local solar system. And is ready to accept a generalization, that this kinematic law will apply to other solar systems of like pattern as well.

Thus, accepting Kepler's first and second law, X acquires a rather clear and distinct idea of an infinite mode of planetary motion as a facet of the idea of the aeternal and infinite mode of **motion and rest**, which is the **attribute of extension**.

A check (not easy, at least I still struggle with it), in how far this little example provides additional support for the interpretational theses of the previous paragraphs, is left to the benevolent reader.

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